

TO THE CALCULATION OF PSEUDO-CONSERVATIVE SELF-OSCILLATION VIBRAIMPACT SYSTEMS

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ABSTRACT

A class of models of self-oscillated vibration (in particular) systems allowing periodic motions accompanied by mutual compensation of excitation and dissipation forces, is considered. The models allow analysis of certain types of shock vibration systems absolutely flexible filaments, and longitudinally vibrating bars excited by the so-called "negative" friction. A technique for calculating a self-excited vibration machine whose model belongs to the above class, is presented.

Keywords: *Self-oscillationys, impacts, strings, vibroimpacts systems.*

The notion of self-oscillated mechanical systems has originally been introduced in [1]. The major principles for designing self-oscillated technological vibration machines, in particular, impact vibration machines, were formulated in [2-4]. The systems are calculated on the basis of the energy-conservation principle according to which the work done by dissipation forces is compensated by the work done by excitation forces during a cycle of motion.

Consider a scleronomic mechanical system with complete dissipation and n degrees of freedom. Let the kinetic and potential energies of the system be represented by symmetric, positive definite matrix $A([n \times n])$ and function $C_0(x) \in \mathbb{R}^n$ respectively, where $x \in \mathbb{R}^n$ is a vector of generalized coordinates. If dissipation and excitation forces are defined by functions $g_1(x, \dot{x})$ and $g_2(x, \dot{x})$ respectively, where $g_1, g_2 \in \mathbb{R}^n$, then the equation of motion has the form

$$Ax + F(x) + g_1(x, \dot{x}) = g_2(x, \dot{x}), \quad F(x) = \text{grad}C_0(x). \quad (1)$$

Then for a T -periodic self-oscillated mode of motion $x_0(t)$, we have

$$\int_0^T [g_2(x, \dot{x}) - g_1(x, \dot{x})] x_0(t) dt = 0 \quad (2)$$

where the integrand contains scalar products of the vectors [4].

In the other case, for the state of the system described by displacement function $u(x, t)$, and nonconservative forces represented in the form $g_{1,2}(u, \dot{u}_i)$, we have

$$\int_X \int_0^T [g_2(u, \dot{u}_i) - g_1(u, \dot{u}_i)] u_i(x, t) dx dt = 0 \quad (3)$$

where $u(x, t) \equiv u(x, t + T)$, and integration with respect to variable x is carried out over domain X .

Relationships of the type of Eqs.(2), and (3) form the basis for the analysis of self-oscillated vibration systems. As a rule, by substituting into them some presupposed representations of the sought modes one can find certain unknown parameters of the laws of motion.

However, for certain systems, Eqs.(2), and (3) allow finding periodic modes for which nonconservative forces can be balanced at any instant. In this case self-oscillated vibration sets in, and the motion proceeds as it would in the absence of friction.

Let us consider system (1). Suppose that for $g_1=g_2=0$ (i.e. for a conservative system) a two-parameter family of T_0 -periodic solutions $x=x_0(A; t - \varphi)$ may be constructed, where A is an integral uniquely related to the total energy. Dependence $A(\omega_0)$ where $\omega_0=2\pi T_0^{-1}$, defines the skeletal curve. Without loss of generality, arbitrary phase φ may be put equal to zero. Suppose that for certain values of $A=A^0$ and $\omega=\omega_0$, relationship (2) for system (1) may be replaced by a stronger one:

$$g_1[x_0(A^0; t), x_{0t}(A^0; t)] = g_2[x_0(A^0; t), x_{0t}(A^0; t)] \quad (4)$$

valid for all $t \in \mathbb{R}$.

Then, it follows from (1) that in line with the above assumption, $Ax_{0tt}(A^0, t) + F[x_0(A^0, t)] = 0$. That is why self-oscillated vibration $x_0(A^0, t)$ whose period is $2\pi\omega^{-1}$, proceeds as if in the absence of friction.

In what follows the systems satisfying conditions (4) are called pseudo-conservative, or systems with fully compensated dissipation forces [5].

This class of systems may be considered as no representative one but we hope we will show that it is not quiet right.

The statement that many vibroimpact systems could be described by means of these models is explicated by the fact that many representations of vibroimpact systems' laws of movement are described by piecewise-linear functions (accurately or approximately).

The famous triangles of Helmholtz [6], which nature was explicated by Vitt [7], appear just because of string set membership of pseudo-conservative systems (see also our work [8] where string vibrating against different obstacles in the entering airflow were researched). We will show that violin strings vibrating against different obstacles could be described as well by means of the same basic models. We could show that vibroimpact technological machines are successfully described, thus we can synthesize various control systems namely the so-called resonant robots [8].

In case of use of other (nonnewton's) hypothesizes of impacts the singularization techniques [4] allows to receive necessary general results.

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