ALMOST PERIODIC OSCILLATIONS IN VIBROIMPACT SYSTEMS

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ABSTRACT

The presence at anyone vibroimpact system large nonlinear positional forces - it is a reason of clear manifestation in them of various nonlinear effects [1]. Inception of almost-periodic processes - it is one from major and also important such effects and these effect is studied. We have two base models there: resonant systems with one degree of freedom and some systems with the distributed impact elements are considered.

1.In this part the small perturbations of conservative vibration impact systems with one degree of freedom are considered. It is suggested that perturbation periodically depends on fast and slow time. Corresponding system in variables impulse-phase have the form

$$\frac{dx}{dt} = \varepsilon\, X(x, y, t, \tau), \qquad \frac{dy}{dt} = \omega\,(x) + \varepsilon\, Y(x, y, t, \tau). \qquad (1)$$

Here ε is a small parameter, $\tau = \varepsilon\, t$ is a slow time, $x(t), y(t)$ are scalar functions. The functions $X(x, y, t, \tau)$ and $Y(x, y, t, \tau)$ are periodic by t with period $2\pi/\nu$ and by τ with period $2\pi/\sigma$. The function $X(x, y, t, \tau)$ has finite discontinuities with respect to y in points $y = 2\pi\, l$ (l - integer number). Qualitative behavior of solutions of system (1) in neighborhood of radius $\sqrt{\varepsilon}$ of individual resonance level $x = x_{pq}$ is investigated. The individual resonance level is defined by relation $\omega\,(x_{pq}) = q\nu/p$ where p,q are relatively prime numbers. For system (1) the averaged system of the second approach by parameter $\sqrt{\varepsilon}$ is considered. Analysis of averaged system make it possible to obtain classification of type of qualitative behavior of solutions of system (1). The principal role play sign of expression

$$\delta(\tau) = \frac{\nu}{2\pi\, p} \int_0^{\frac{2\pi p}{\nu}} \left[X_x(x_{pq}, y + q\nu t/p, t, \tau) + Y_y(x_{pq}, y + q\nu t/p, t, \tau) \right] dt\,.$$

The theorems of existence and stability the almost periodic solutions of system (1) are obtained. For example, the equation

$$x'' + \Omega^2 x + \Phi(x, x') = \varepsilon \left[\gamma\, x' + a_1 \sin \nu t + a_2 \sin(\nu t + \varepsilon\sigma t)\right] \quad (2)$$

is considered. Here $\Phi(x, x')$ is the force of elastic impact, $\gamma > 0, \Omega, a_1, a_2, \nu, \sigma$ are real constants. The equation (2) describes the forced oscillations of a sum of small periodic functions with close frequencies.

2. In many cases oscillations in vibroimpact systems with distributed impact elements, e.g. strings vibrating near various obstacles (straight, pointwise, etc.) [2, 3] are described with the help of the formulas similar such: *u(x,t)=A[y(x,t),τ(x,t)].* Here *u* is state function of distributed system (e.g. is deflection of string); *A(y,τ)* – some periodic function: $A(y,\tau+T_0)\equiv A(y,\tau)$;), *τ(x,t)* –other periodic function: $\tau(x,t+T_1)\equiv \tau(x,t)$. Periods T_0 и T_1 depend on physical and geometric parameters of our system, and if T_0 и T_1 are incommensurable numbers than *u(x,t)* is almost periodic function and we have almost periodic vibroimpact process.

Different cases of arising the similar processes in systems with simple and complex structures are studied.

Some of theoretical rezults and a certain experimental data are compared.

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